Incorporating waveform uncertainty into modeling and inference of GWs

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Introduction

Uncertainties in waveform models

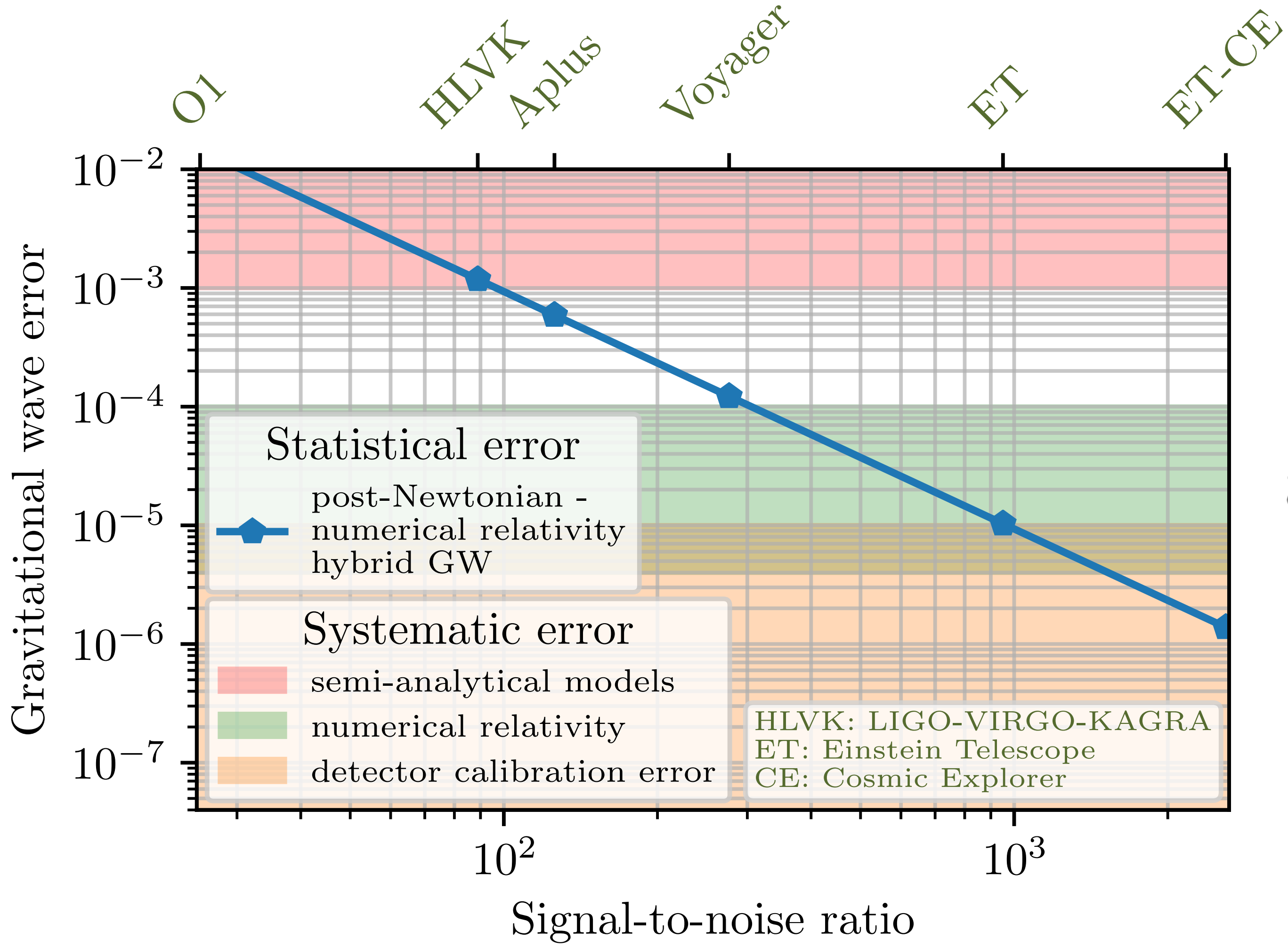
- Errors come from:
  - **inputs** (PN, EOB, NR)
  - **modeling error** (fits, interpolation, basis construction)
  - **simplifications in physical modeling** (e.g. reference frame for precession, ignoring eccentricity, asymmetry in modes)

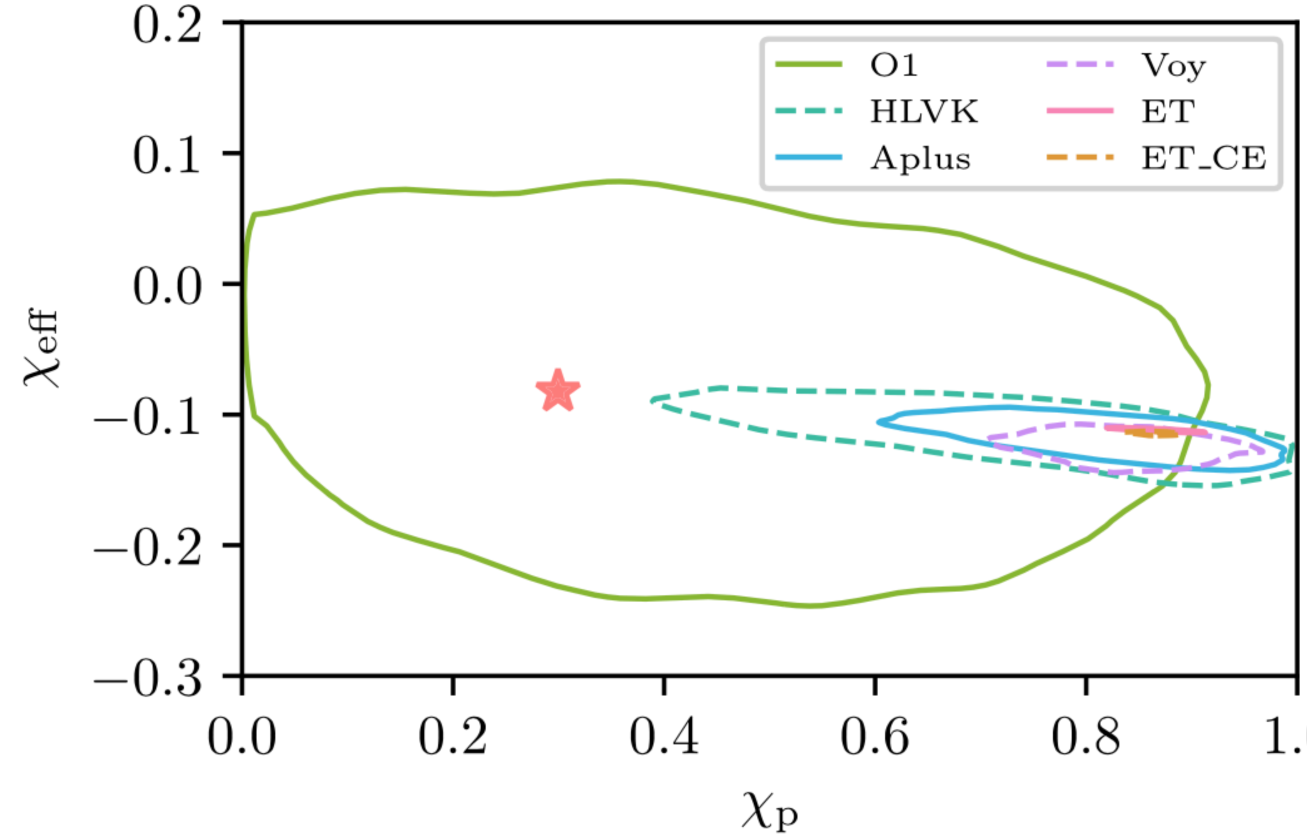
- How big are these errors?
  - **Mismatch** (unfaithfulness) can reach **several %** for semi-analytical models; smaller for NR
  - Depends on how **"extreme"** the configuration is.

Systematic errors in inference

- **Systematic errors in waveform models can manifest as**
  - **parameter biases** when models are used for inference of binary parameters
  - **false positives for tests of GR** (e.g. nonzero residual)
- **Importance of systematic errors depends on signal-to-noise ratio and "extremeness" of binary**
  - Waveform systematics **subdominant** for e.g. GW150914
  - Waveform systematics will **dominate** over statistical error as we approach the much more sensitive **3rd generation ground-based GW detectors** (Einstein Telescope & Cosmic Explorer)

Example: Measuring GW150914-like binaries





[MP & C. Haster, Phys.Rev.Res. 2 (2020) 2, 023151]

- **Statistical error** falls off with SNR
- **Systematic error** is constant
- Expect biases if stat <~ sys error

- **Bias** in effective precession spin
  - Signal: NR-PN hybrid
  - Template: IMRPhenomPv2

How to mitigate waveform systematics

- Build **more accurate models**
  - requires more & better NR simulations,
  - more accurate analytical information (PN, EOB, SF)
- Build **models that incorporate error estimates**
  - additional degrees of freedom can parameterize modeling error and errors in inputs
  - use in inference and marginalize over error parameters

Work on incorporating waveform errors

- C. Moore & J. Gair, PRL 113, 2014:
  - analytically marginalize **waveform uncertainty** $\delta h(\lambda) = h_{\text{approx}}(\lambda) - h_{\text{accurate}}(\lambda)$ with a prior distribution constructed by using **Gaussian process regression**
  - Application: 1D PN toy model in chirp mass

- C. Moore et al, PRD 93, 2016:
  - IMRPhenomC / TaylorF2, 1D in chirp mass

- P. Landry et al, PRD 99, 2019:
  - NS EOS

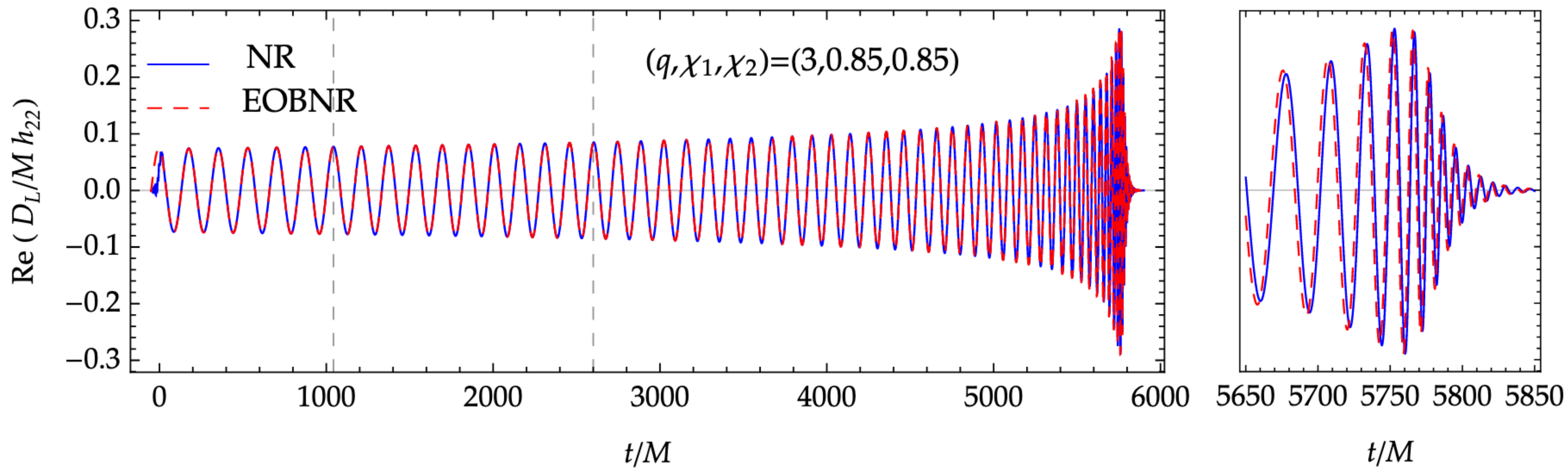
- A. Chua et al, PRD 101, 2020:
  - LISA/EMRIs

- B. Edelman et al, 2020:
  - Spline model to parametrize waveform deviations
  - See also: W. Farr et al, LIGO- T1400682 for handling detector calibration errors in PE

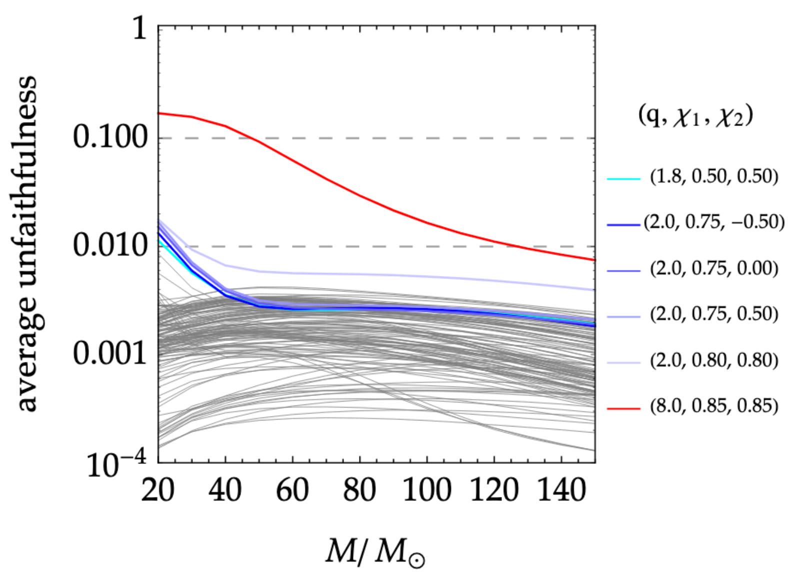
A model for waveform uncertainty in SEOBNRv4

The SEOBNRv4 model

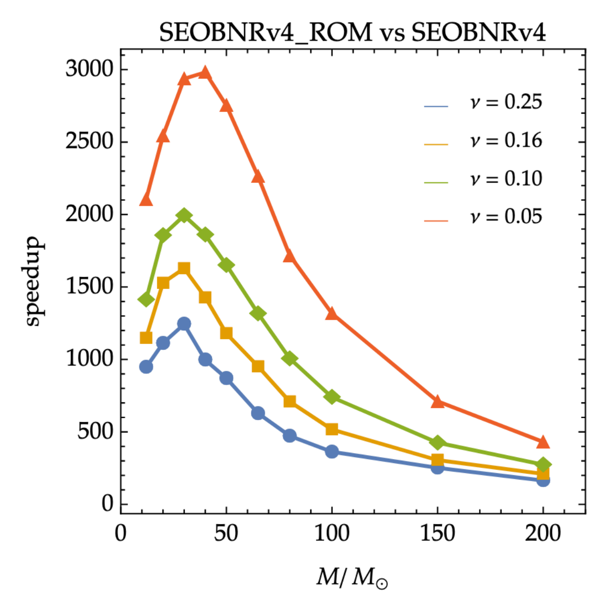
Effective-one-body model for binaries with **non-precessing spins** (Bohé et al, PRD 95, 2017)



Accuracy vs NR



Reduced order model



SEOBNRv4_ROM
follows method in
MP, CQG 31, 2014
MP, PRD 93, 2016

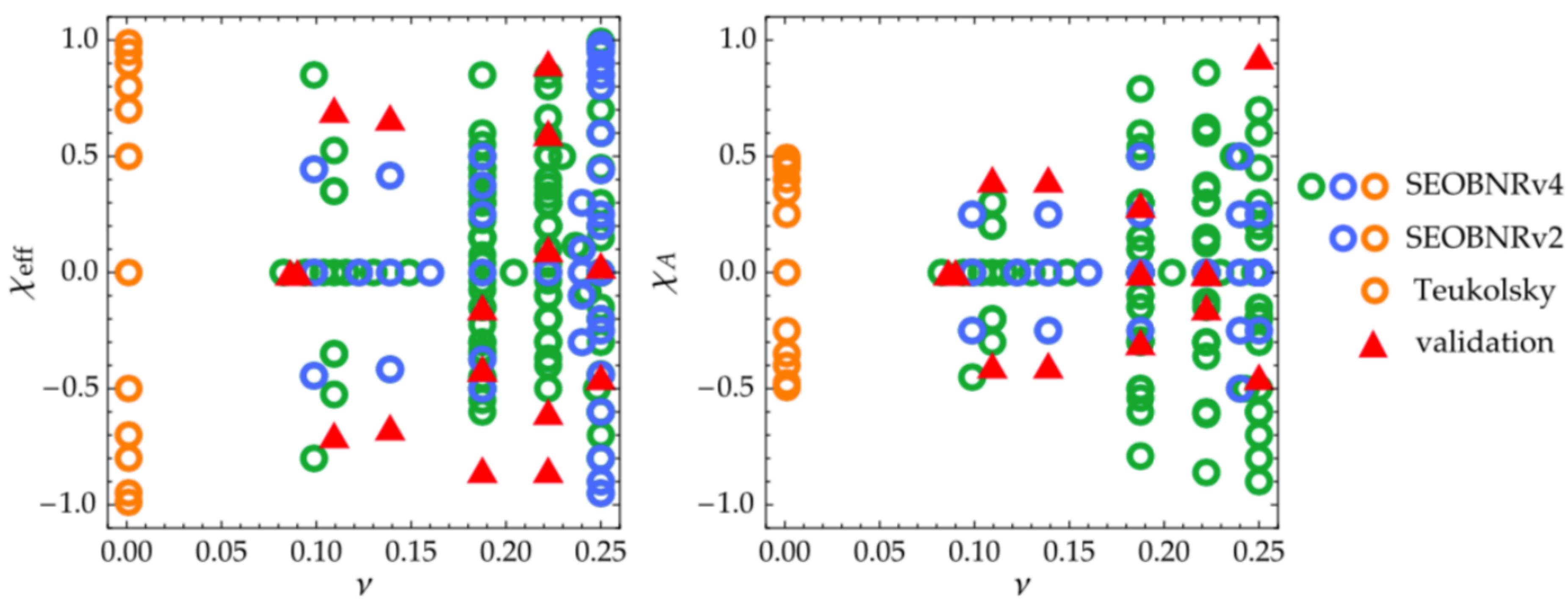
The SEOBNRv4 model

- **The uncalibrated model** depends on **physical parameters** λ and **calibration parameters** θ:

$$h_{\mathrm{EOB}}(\lambda, \theta)$$

$$\lambda = (q, \chi_1, \chi_2) \qquad \theta = (K, d_{\mathrm{SO}}, d_{\mathrm{SS}}, \Delta t^{22}_{\mathrm{peak}})$$

- **Calibration** fixes $\boldsymbol{\theta}(\boldsymbol{\lambda})$ based on match between EOB waveforms and NR at 141 specific points $\{\boldsymbol{\lambda}_i\}$.



The SEOBNRv4 model

- Step 1: **For each λ_i**, determine **"calibration posterior"** over $\boldsymbol{\theta}$ satisfying a figure of merit:
  a. MCMC to obtain samples from

mismatch vs NR

difference of merger time vs NR

$$P(\boldsymbol{\theta}) \propto \exp\left[-\frac{1}{2}\left(\frac{\mathfrak{M}_{\max}(\boldsymbol{\theta})}{\sigma_{\mathfrak{M}}}\right)^2 - \frac{1}{2}\left(\frac{\delta t^{22}_{\mathrm{peak}}(\boldsymbol{\theta})}{\sigma_{\delta t}}\right)^2\right]$$

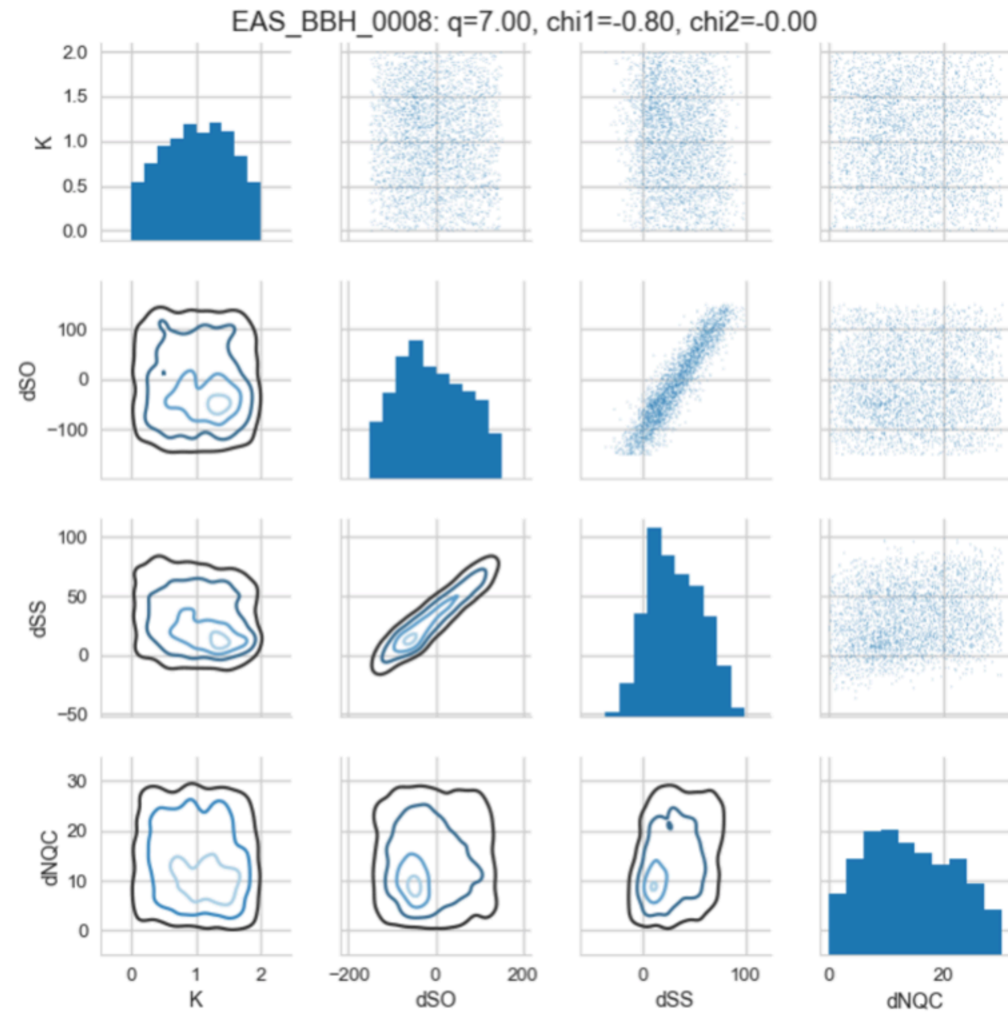
1% **5M**

  b. discard $\boldsymbol{\theta}$ with $\mathfrak{M}_{\max} > 1\,\%$ and $|\delta t^{22}_{\mathrm{peak}}| > 5M$
  c. remove secondary modes from distribution by hand

$$\mathfrak{M}(h_1, h_2) := 1 - \mathcal{O}(h_1, h_2) \qquad \mathcal{O}(h_1, h_2) = \frac{4}{\|h_1\|\,\|h_2\|} \max_{t_0} \left| \mathcal{F}^{-1}\left[\frac{\tilde{h}_1(f)\tilde{h}_2(f)^*}{S_n(f)}\right](t_0) \right|$$

The SEOBNRv4 model

- Example calibration posterior at an NR calibration point:



- Construct **polynomial fit to means** $\bar{\boldsymbol{\theta}}(\lambda_i)$ to obtain $\boldsymbol{\theta}(\lambda)$ throughout physical parameter space.

- Find $\mathfrak{M}_{\max} < 1\,\%$ against NR waveforms.

Modeling the uncertainty

- We want to incorporate the **uncertainty in the waveform model** into **parameter estimation**
  $h(\boldsymbol{\lambda}) \to p(h \,|\, \boldsymbol{\lambda})$

- **Method:**
  1. At each NR calibration point $\boldsymbol{\lambda}_i$, determine **parametrized distribution** $p(h \,|\, \lambda)$
  2. **Interpolate the distributions** to all $\boldsymbol{\lambda}$
  3. Work with **amplitude & phase deviations** for efficiency.

Calibration error (CE) model ingredients

1. **Compressed frequency-domain representation of waveform "error":**
   - **Amplitude and phase deviations** from SEOBNRv4,

$$h = \left[1 + \delta A(f)\right] e^{i\delta\phi(f)} \times h_{\text{SEOBNRv4}}$$

   - log-spaced frequency nodes,

$$\delta A_j = \delta A(f_j), \quad \delta\phi_j = \delta\phi(f_j), \quad j = 1,\ldots,10$$

2. **Multivariate normal approximation**

$$p\left((\delta A, \delta\phi)\,|\,\boldsymbol{\lambda}_i\right) \approx \mathcal{N}\left(\mu(\boldsymbol{\lambda}_i), \Sigma(\boldsymbol{\lambda}_i)\right)$$

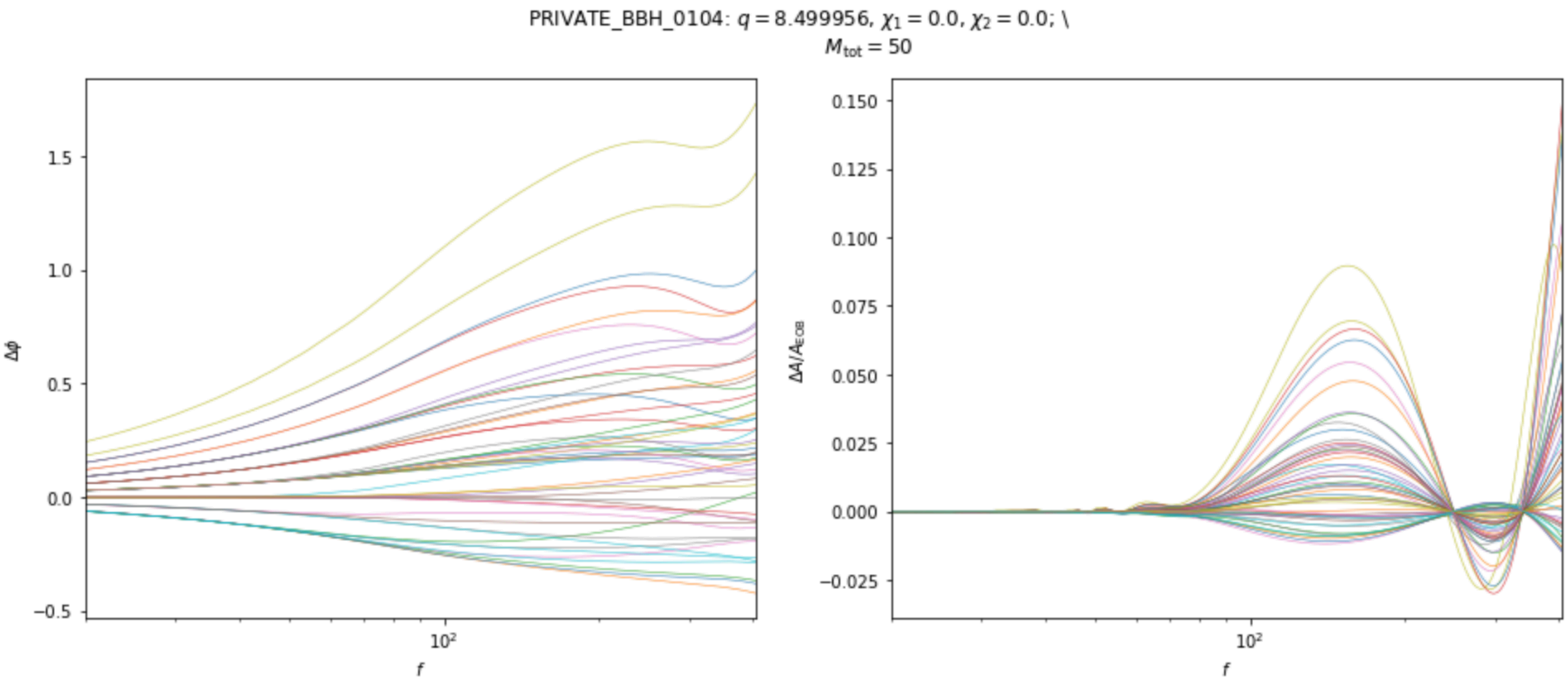
3. **GPR interpolation**

$$\left(\mu(\boldsymbol{\lambda}_i), \Sigma(\boldsymbol{\lambda}_i)\right) \mapsto \left(\mu(\boldsymbol{\lambda}), \Sigma(\boldsymbol{\lambda})\right)$$

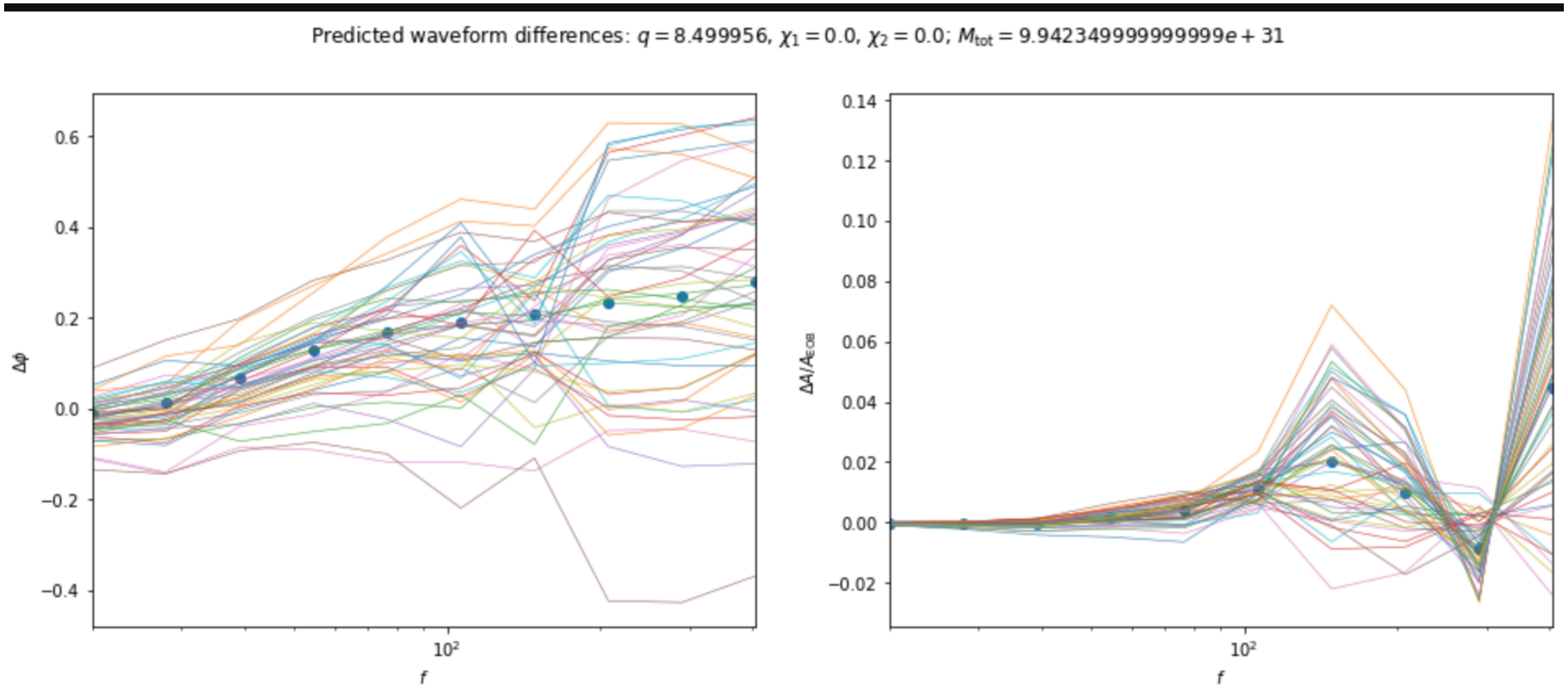
- One can then **rapidly draw** $\boldsymbol{\epsilon} := (\{\delta A_i\}, \{\delta\phi_i\}) \sim p((\delta A, \delta\phi)\,|\,\boldsymbol{\lambda})$, apply this correction to SEOBNRv4_ROM waveforms, and perform inference.

Example waveform distribution at NR point (validation)

actual waveform differences

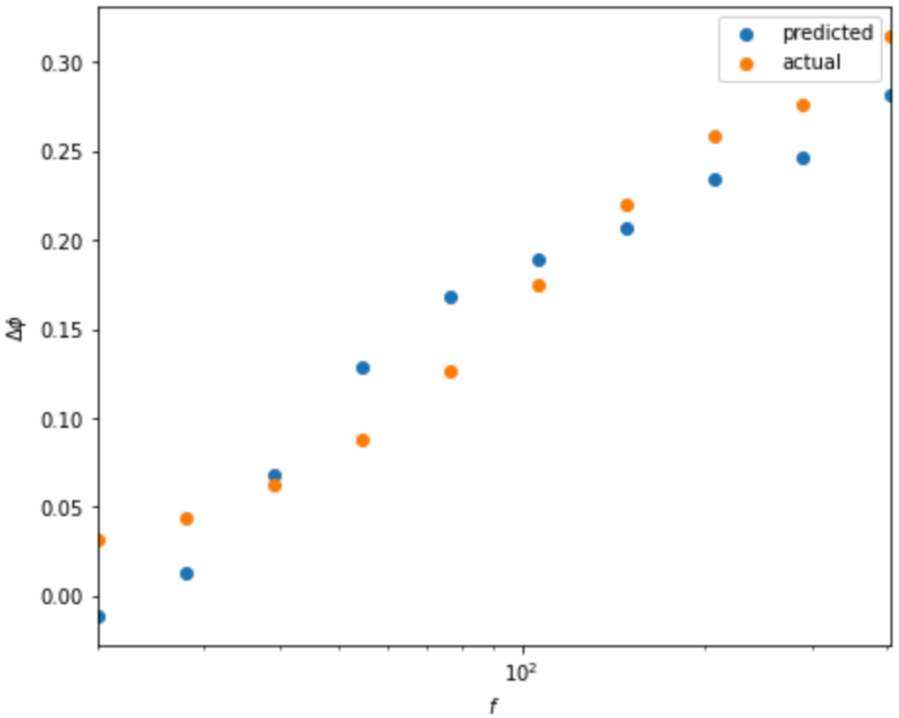


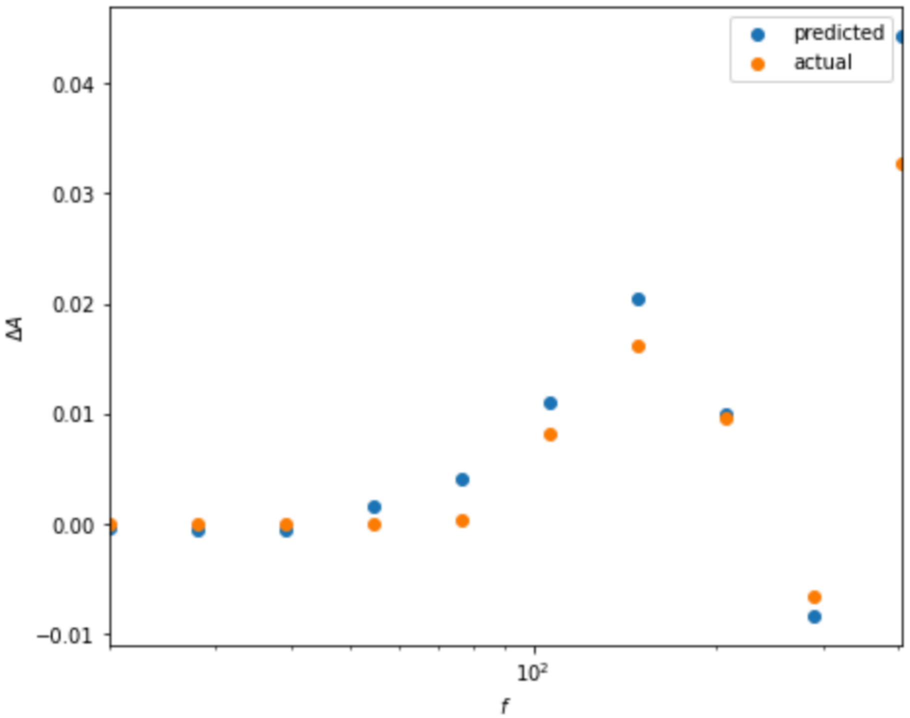
model waveform differences



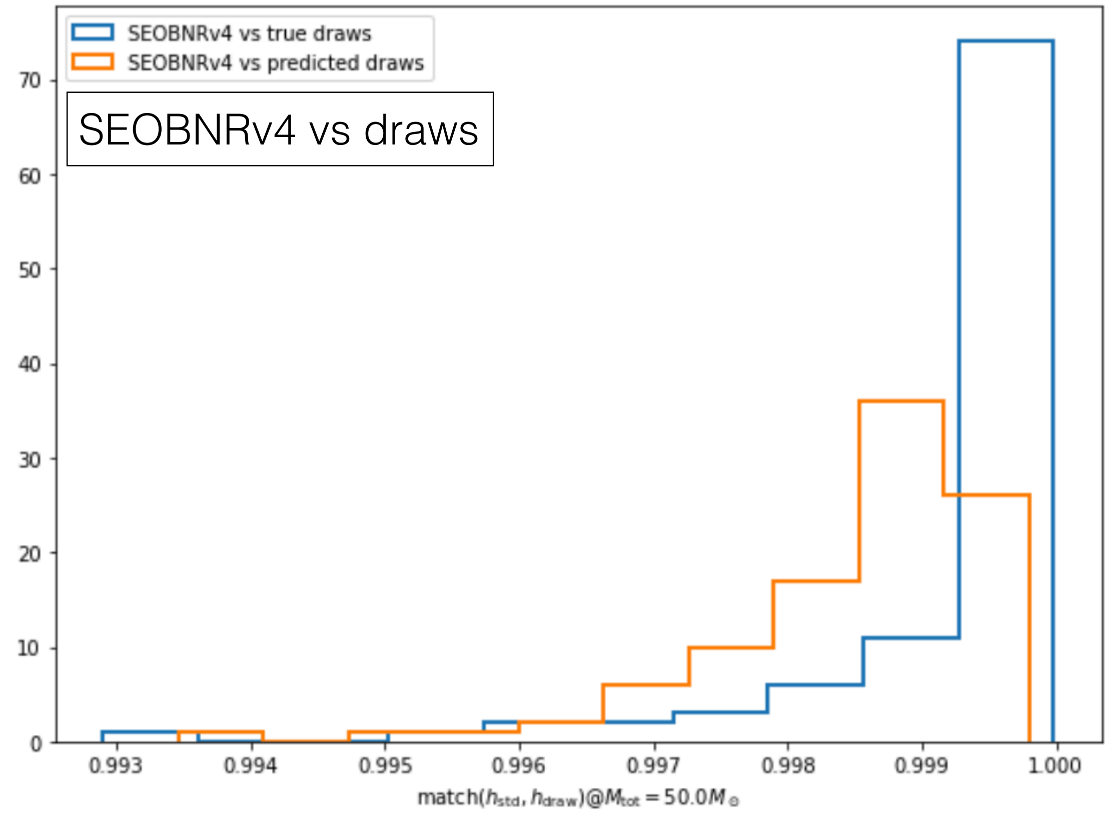
Example CE accuracy at NR point (validation)

Means comparison (predicted vs actual)





Match (predicted vs actual)



Summary: SEOBNRv4 calibration error model

- CE model:
  - Data: **SEOBNRv4 calibration posteriors against NR**
  - **Amplitude and phase deviations** w.r.t. a neutral EOB calibration (calibration means) modeled by GPRs
  - parameterized by **2 x 10 additional parameters $\boldsymbol{\epsilon}$** for amplitude and phase

$$\delta\tilde{h}_{CE}(\boldsymbol{\lambda}, \boldsymbol{\epsilon}; f) = (1 + \delta A(\boldsymbol{\lambda}, \boldsymbol{\epsilon}; f)) \exp(i\,\delta\phi(\boldsymbol{\lambda}, \boldsymbol{\epsilon}; f))$$

- Combine error model with base waveform:
  - `SEOBNRv4CE` := `SEOBNRv4_ROM` * `CE` model
  - `SEOBNRv4CE0`: Neutral ($\boldsymbol{\epsilon} = 0$) calibration != `SEOBNRv4_ROM`

$$\tilde{h}_{CE}(\boldsymbol{\lambda}, \boldsymbol{\epsilon}; f) = \tilde{h}_{\mathrm{SEOBNRv4\_ROM}}(\boldsymbol{\lambda}; f)\; \delta\tilde{h}_{CE}(\boldsymbol{\lambda}, \boldsymbol{\epsilon}; f)$$

Parameter estimation study

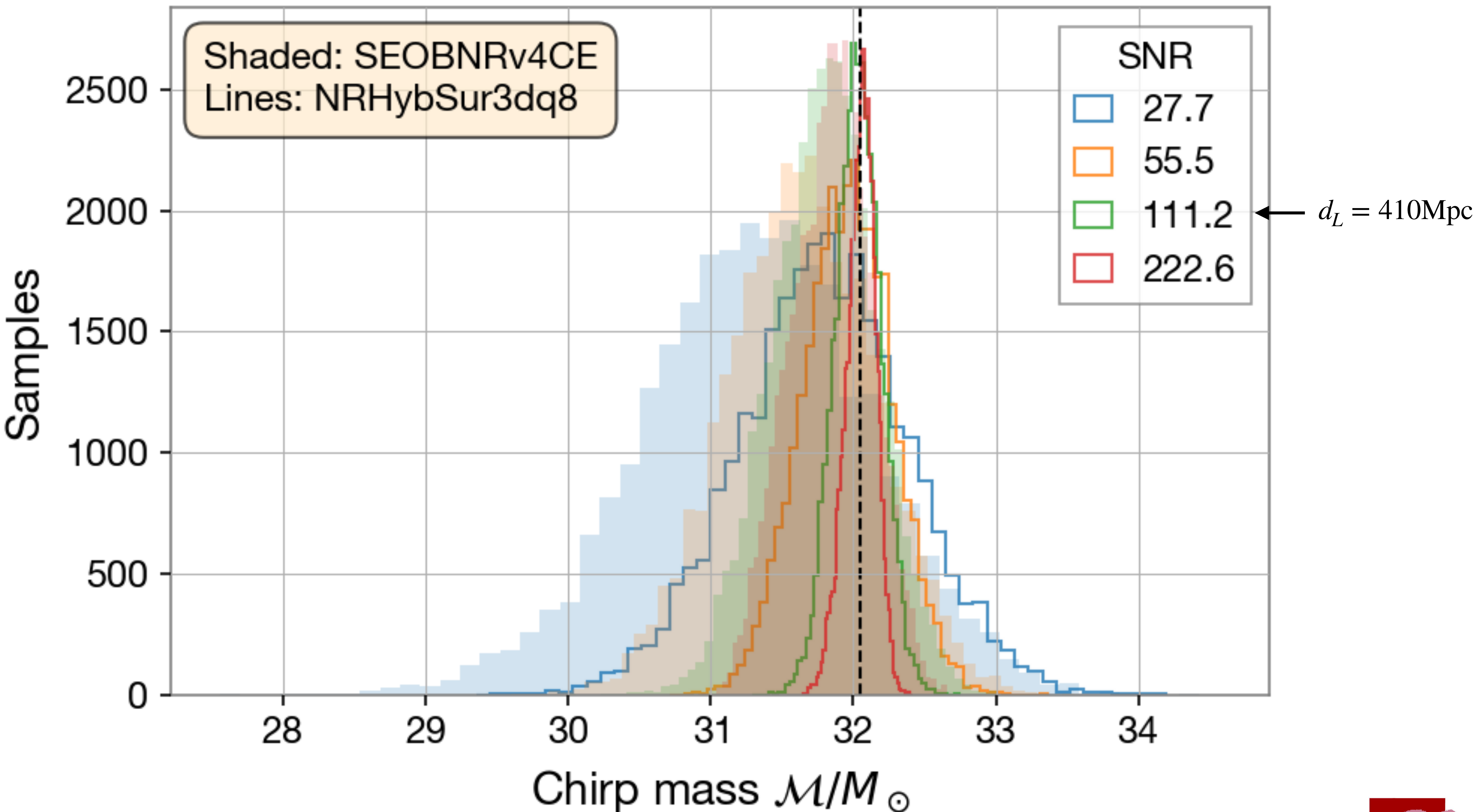
Parameter estimation of synthetic BBH signals

- **PE Setup:**
  - GW150914-like signal parameters
  - LIGO-Virgo (HLV) design sensitivity, varying distance / SNR, zero noise
  - PE with **bilby** & **dynesty** [https://git.ligo.org/lscsoft/bilby]
- **Sampling and marginalizing over waveform uncertainty:**
  - Inference over standard waveform parameters and CE parameters $\boldsymbol{\epsilon}$
  - Use zero-mean Gaussian priors for CE parameters
  - Marginalize posterior over CE parameters

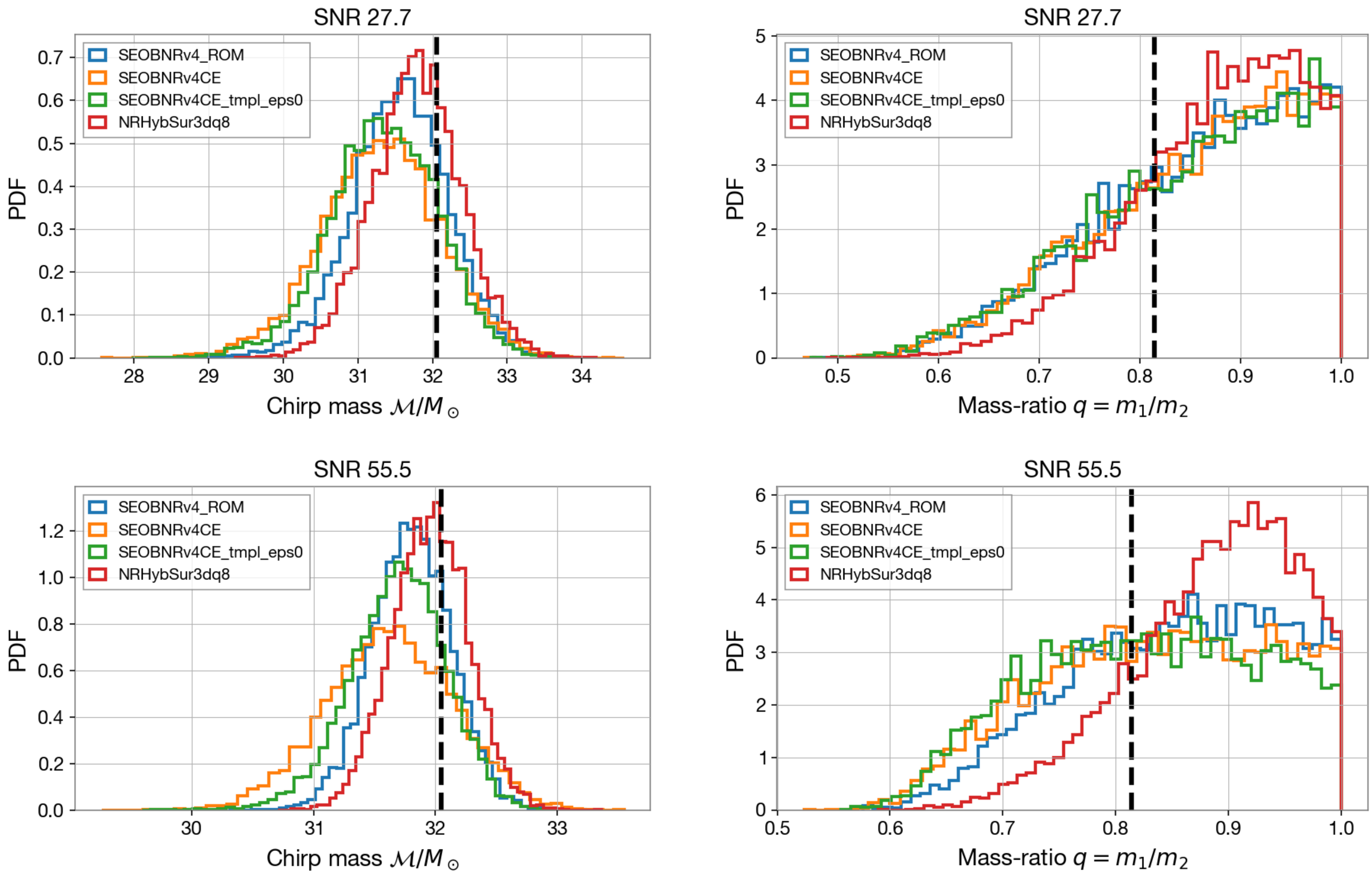
See W. Farr et al, LIGO- T1400682

- **NRSur3dq8 signal** recovered with **waveform models**:
  - `SEOBNRv4_ROM`
  - `SEOBNRv4CE`: sampling in $\boldsymbol{\lambda}$ and $\boldsymbol{\epsilon}$
  - `SEOBNRv4CE0` with $\boldsymbol{\epsilon} = 0$
  - `NRSur3dq8`

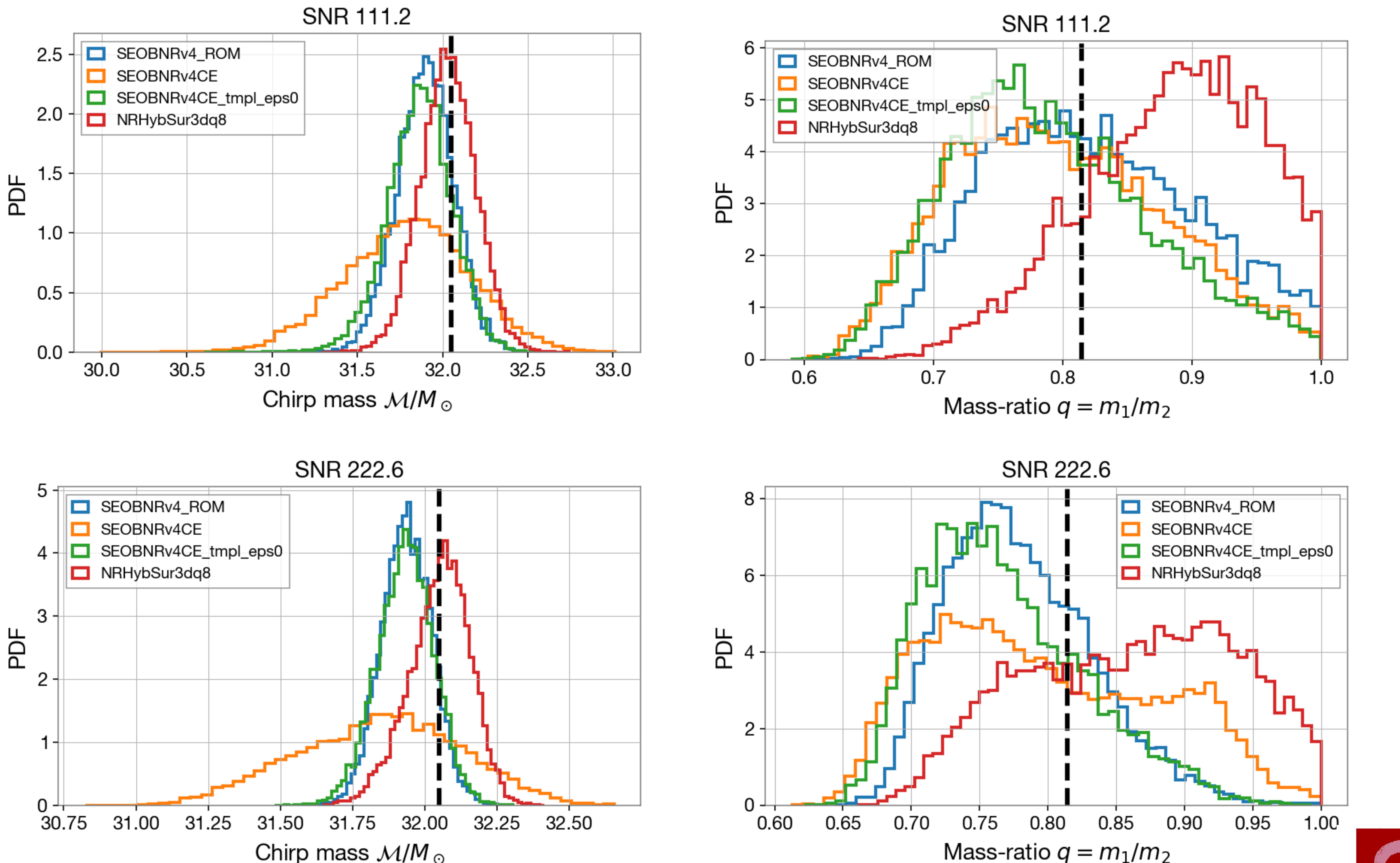
Evolution of chirp mass with SNR



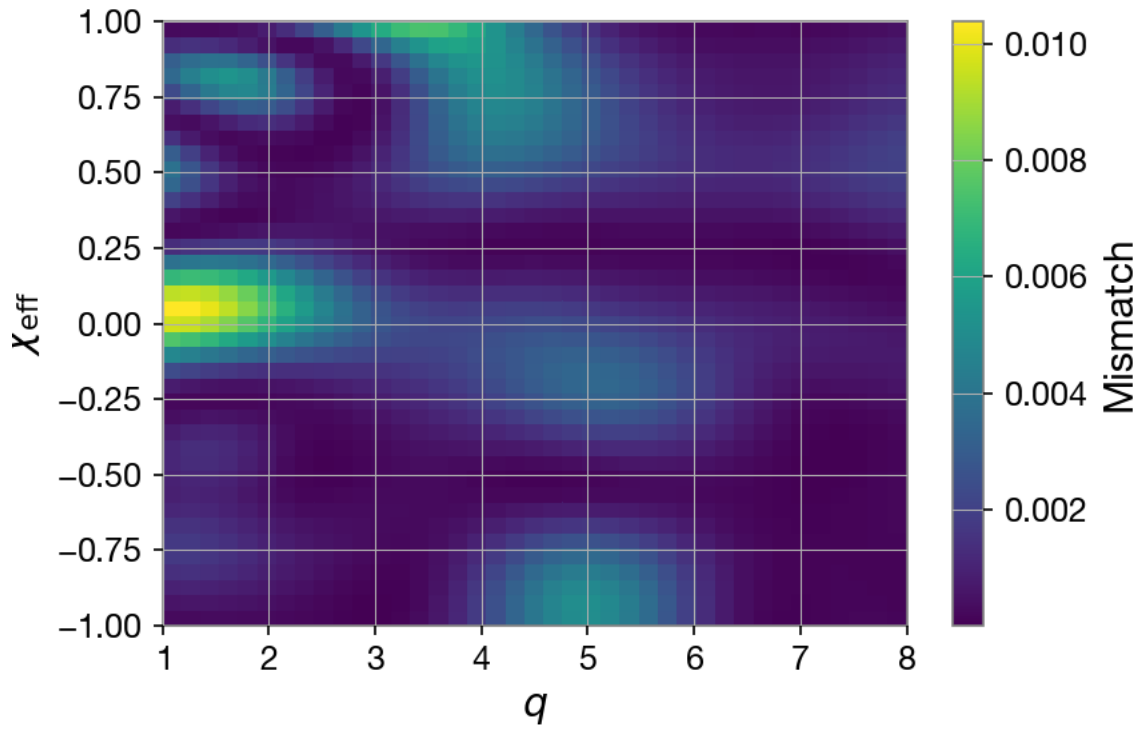
Marginal posterior distributions



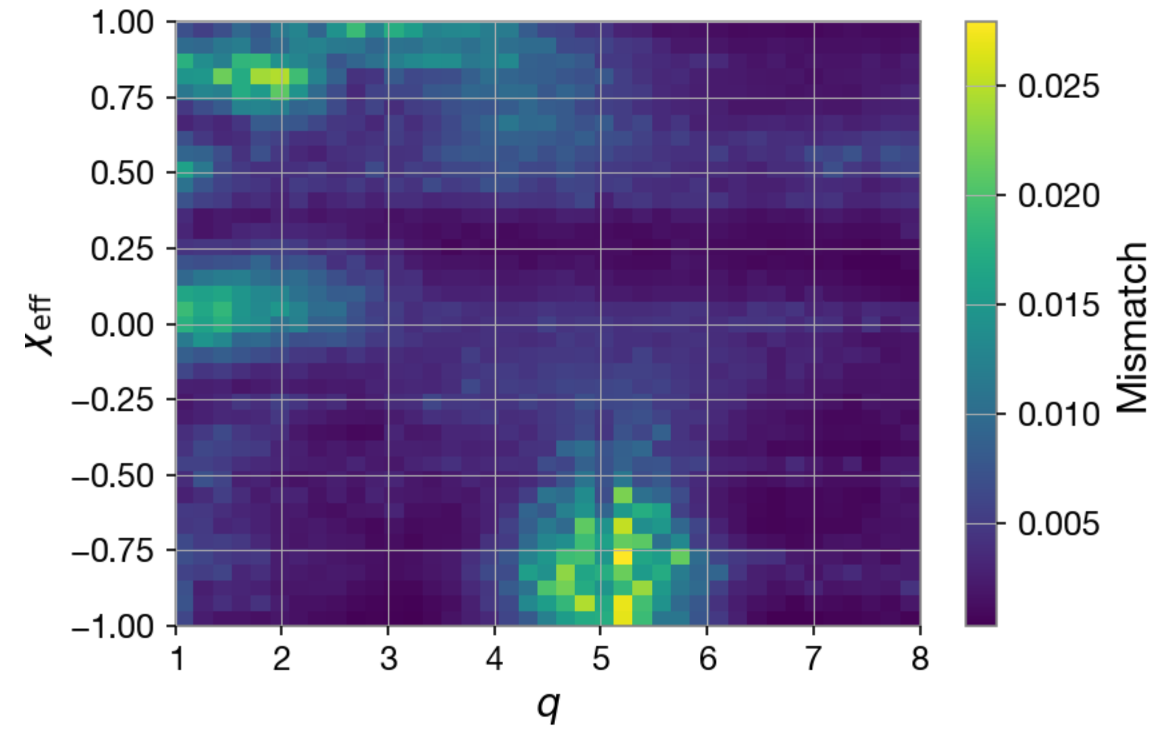
Marginal posterior distributions



Mismatch against SEOBNRv4_ROM



$\mathfrak{M}_{\mathrm{CE}}(\boldsymbol{\epsilon} = 0)$



95th percentile of $p(\mathfrak{M}_{\mathrm{CE}})$

- **Mismatch distribution over binary parameter space:**

$$p(\mathfrak{M}_{\mathrm{CE}}(\boldsymbol{\epsilon};\boldsymbol{\lambda})\,|\,\boldsymbol{\epsilon} \sim \mathcal{N}(0,1);\boldsymbol{\lambda})$$

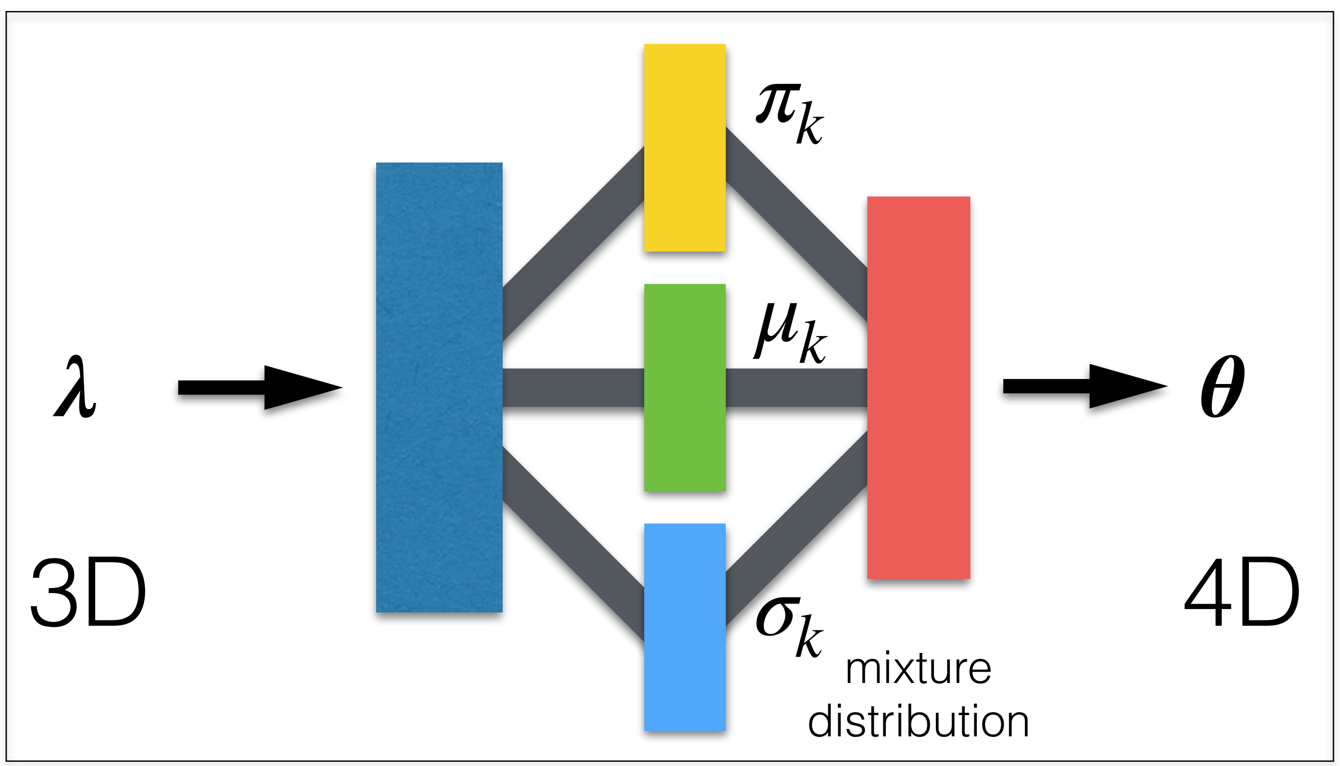
$$\mathfrak{M}_{\mathrm{CE}}(\boldsymbol{\epsilon};\boldsymbol{\lambda}) := \mathfrak{M}(\text{SEOBNRv4CE}, \text{SEOBNRv4\_ROM})(\boldsymbol{\epsilon};\boldsymbol{\lambda})$$

- **Isolated regions:** EOB fit deviates from mean of calibration posteriors $\bar{\boldsymbol{\theta}}(\boldsymbol{\lambda}_i)$

MDN for calibration posteriors

- SEOBNRv4: **polynomial fit to the means of the calibration posteriors** $p(\boldsymbol{\theta}\,|\,\boldsymbol{\lambda})$ at the NR points $\{\boldsymbol{\lambda}_i\}$
- We can **improve** on this by a using **mixture of Gaussians**

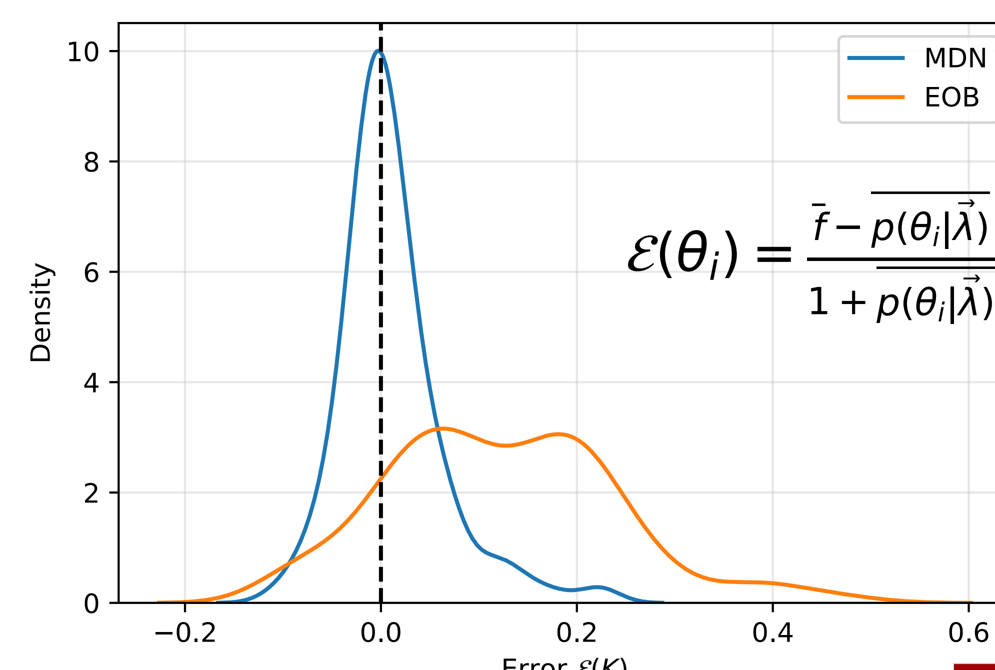
$$p(\boldsymbol{\theta}\,|\,\boldsymbol{\lambda}) \approx \sum_{k=1}^{K} \pi_k(\boldsymbol{\lambda})\mathcal{N}(\boldsymbol{\theta}\,|\,\mu_k(\boldsymbol{\lambda}), \sigma_k^2(\boldsymbol{\lambda}))$$



Mixture density network

Validation

$$D_{KL}(p(\boldsymbol{\theta}\,|\,\boldsymbol{\lambda}_i), \mathrm{MDN}) \approx 0.5\mathrm{bits}$$



Mean errors

Conclusion

- Built **efficient GPR uncertainty model** SEOBNRv4CE:
  - Parametrizes EOB calibration uncertainty against NR
  - Works as expected for parameter inference and marginalizing over error parameters!
  - Posteriors PDFs are **less precise**, but **reduce biases**.
- Have **improved upon the calibration fit** used in SEOBNRv4
  - Mixture density network model of $p(\boldsymbol{\theta} \mid \boldsymbol{\lambda})$
- Work on **7D joint model for "uncalibrated EOB"** $h(\boldsymbol{\lambda}; \boldsymbol{\theta}; f)$
  - Challenging modeling problem!
  - Why? $\boldsymbol{\theta}$ has *physical interpretation*, whereas $\boldsymbol{\epsilon}$ is more *phenomenological*
  - Could combine with MDN $p(\boldsymbol{\theta} \mid \boldsymbol{\lambda})$ and obtain posteriors for $\boldsymbol{\theta}$
  - If GR value for $\boldsymbol{\theta}$ known: can test GR

Thank you for your attention!